

**ALGEBRAIC PROPERTIES OF EQUALITY**

Let  $a$ ,  $b$ , and  $c$  be real numbers.

<b>ADDITION PROPERTY</b>	If $a = b$ , then $a + c = b + c$ .
<b>SUBTRACTION PROPERTY</b>	If $a = b$ , then $a - c = b - c$ .
<b>MULTIPLICATION PROPERTY</b>	If $a = b$ , then $ac = bc$ .
<b>DIVISION PROPERTY</b>	If $a = b$ and $c \neq 0$ , then $a \div c = b \div c$ .
<b>REFLEXIVE PROPERTY</b>	For any real number $a$ , $a = a$ .
<b>SYMMETRIC PROPERTY</b>	If $a = b$ , then $b = a$ .
<b>TRANSITIVE PROPERTY</b>	If $a = b$ and $b = c$ , then $a = c$ .
<b>SUBSTITUTION PROPERTY</b>	If $a = b$ , then $a$ can be substituted for $b$ in any equation or expression.

**Match the statement with the Property of Congruence.**

- |  |                        |
|--|------------------------|
| 1. For any segment $\overline{XY}$ , $\overline{XY} \cong \overline{XY}$   | A. Transitive Property |
| 2. If $\overline{JK} \cong \overline{MN}$ and $\overline{MN} \cong \overline{CD}$ , then $\overline{JK} \cong \overline{CD}$ . | B. Symmetric Property  |
| 3. If $\overline{BN} \cong \overline{TR}$ , then $\overline{TR} \cong \overline{BN}$ .   | C. Reflexive Property  |

**Skill Check** ✓ In Exercises 4–8, match the conditional statement with the property of equality.

- |  |                            |
|--|----------------------------|
| 4. If $JK = PQ$ and $PQ = ST$ , then $JK = ST$ .                       | A. Addition property       |
| 5. If $m\angle S = 30^\circ$ , then $5^\circ + m\angle S = 35^\circ$ . | B. Substitution property   |
| 6. If $ST = 2$ and $SU = ST + 3$ , then $SU = 5$ .                     | C. Transitive property     |
| 7. If $m\angle K = 45^\circ$ , then $3(m\angle K) = 135^\circ$ .       | D. Symmetric property      |
| 8. If $m\angle P = m\angle Q$ , then $m\angle Q = m\angle P$ .         | E. Multiplication property |

**Match the statement with the property.**

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|--|--|
| 13. If $m\angle S = 45^\circ$ , then $m\angle S + 45^\circ = 90^\circ$ . | A. Symmetric property of equality      |
| 14. If $UV = VW$ , then $VW = UV$ .                                      | B. Multiplication property of equality |
| 15. If $AE = EG$ and $EG = JK$ , then $AE = JK$ .                        | C. Addition property of equality       |
| 16. If $m\angle K = 9^\circ$ , then $3(m\angle K) = 27^\circ$ .          | D. Transitive property of equality     |

Match each statement with the appropriate property of equality.

1) If  $a = b$  and  $c \neq 0$ , then  $\frac{a}{c} = \frac{b}{c}$ .

2) If  $a = b$ , then  $a + c = b + c$ .

3) If  $a = b$ , then  $b = a$ .

4) If  $a = b$ , then  $a \cdot c = b \cdot c$ .

5) If  $a = b$ , then  $a - c = b - c$ .

6)  $a = a$

7) If  $a = b$  and  $b = c$ , then  $a = c$ .

a) Addition Property

b) Subtraction Property

c) Multiplication Property

d) Division Property

e) Reflexive Property

f) Symmetric Property

g) Transitive Property

Justify each statement with a property from algebra or definition or postulate from geometry.

8) If  $AB = CD$  and  $BC = BC$ , then  $AB + BC = CD + BC$

9) If  $3m\angle 1 = 93$  then  $m\angle 1 = 31$

10)  $\overline{BC} \cong \overline{BC}$

11) If point  $A$  is on the interior of  $\angle XOY$ , then  $m\angle XO A + m\angle AO Y = m\angle XO Y$

12) If  $m\angle 1 = m\angle 3$  and  $m\angle 2 = m\angle 3$ , then  $m\angle 1 = m\angle 2$ .

13) If  $\overline{AB} \cong \overline{CD}$  and  $\overline{CD} \cong \overline{EF}$ , then  $\overline{AB} \cong \overline{EF}$ .

**Practice A**

For use with pages 96–101

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58**Match the statement with the Property of Equality.**

- |  |                            |
|--|----------------------------|
| 1. If $JK = PQ$ and $PQ = ST$ , then $JK = ST$ .                       | A. Addition property       |
| 2. If $m\angle S = 30^\circ$ , then $5^\circ + m\angle S = 35^\circ$ . | B. Reflexive property      |
| 3. If $AB + CD = EF + CD$ , then $AB = EF$ .                           | C. Substitution property   |
| 4. $AB = AB$   | D. Transitive property     |
| 5. If $ST = 2$ , then $ST + TU = 2 + TU$ .                             | E. Symmetric property      |
| 6. If $m\angle K = 45^\circ$ , then $3(m\angle K) = 135^\circ$ .       | F. Multiplication property |
| 7. If $m\angle P = m\angle Q$ , then $m\angle Q = m\angle P$ .         | G. Subtraction property    |

**In Exercises 8–13, use the property to complete the statement.**

8. Addition property of equality: If  $AB = 5$ , then  $10 + AB = \underline{\quad ? \quad}$ .
9. Multiplication property of equality: If  $m\angle C = 30^\circ$ , then  $\underline{\quad ? \quad} (m\angle C) = 15^\circ$ .
10. Reflexive property of equality:  $AF = \underline{\quad ? \quad}$ .
11. Symmetric property of equality: If  $m\angle DCF = m\angle MJC$ , then  $\underline{\quad ? \quad}$ .
12. Transitive property of equality: If  $YZ = DB$  and  $\underline{\quad ? \quad} = JK$ , then  $\underline{\quad ? \quad}$ .
13. Substitution property of equality: If  $MN = 3$ , then  $5(MN) = \underline{\quad ? \quad}$ .

**Complete the argument, giving a reason for each step.**

14.  $3(2x - 4) = 5x + 2$       Given

$6x - 12 = 5x + 2$       a.  $\underline{\quad ? \quad}$

$x - 12 = 2$       b.  $\underline{\quad ? \quad}$

$x = 14$       c.  $\underline{\quad ? \quad}$

15.  $4x + 8 = 2x - 12$       Given

$2x + 8 = -12$       a.  $\underline{\quad ? \quad}$

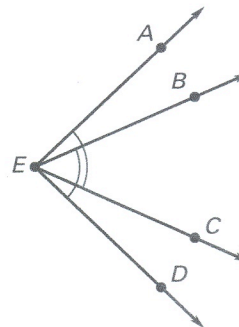
$2x = -20$       b.  $\underline{\quad ? \quad}$

$x = -10$       c.  $\underline{\quad ? \quad}$

16.  $m\angle AEB + m\angle BEC = m\angle CED + m\angle BEC$       Given

$m\angle BEC = m\angle BEC$       a.  $\underline{\quad ? \quad}$

$m\angle AEB = m\angle CED$       b.  $\underline{\quad ? \quad}$

**In Exercises 17 and 18, solve the equation and state a reason for each step.**

17.  $5(2x - 1) = 9x + 4$
18.  $-4(x - 5) = 13$



## Study Guide

## Preparing for Two-Column Proofs

Many rules from algebra are used in geometry.

Properties of Equality for Real Numbers	
Reflexive Property	$a = a$
Symmetric Property	If $a = b$ , then $b = a$ .
Transitive Property	If $a = b$ and $b = c$ , then $a = c$ .
Addition Property	If $a = b$ , then $a + c = b + c$ .
Subtraction Property	If $a = b$ , then $a - c = b - c$ .
Multiplication Property	If $a = b$ , then $a \cdot c = b \cdot c$ .
Division Property	If $a = b$ and $c \neq 0$ , then $\frac{a}{c} = \frac{b}{c}$ .
Substitution Property	If $a = b$ , then $a$ may be replaced by $b$ in any equation or expression.
Distributive Property	$a(b + c) = ab + ac$

**Example:** Prove that if  $4x - 8 = -8$ , then  $x = 0$ .

**Given:**  $4x - 8 = -8$

**Prove:**  $x = 0$

**Proof:**

Statements	Reasons
a. $4x - 8 = -8$	a. Given
b. $4x = 0$	b. Addition Property (=)
c. $x = 0$	c. Division Property (=)

**Name the property that justifies each statement.**

1. Prove that if  $\frac{3}{5}x = -9$ , then  $x = -15$ .

**Given:**  $\frac{3}{5}x = -9$

**Prove:**  $x = -15$

**Proof:**

Statements	Reasons
a. $\frac{3}{5}x = -9$	a. _____
b. $3x = -45$	b. _____
c. $x = -15$	c. _____

2. Prove that if  $3x - 2 = x - 8$ , then  $x = -3$ .

**Given:**  $3x - 2 = x - 8$

**Prove:**  $x = -3$

**Proof:**

Statements	Reasons
a. $3x - 2 = x - 8$	a. _____
b. $2x - 2 = -8$	b. _____
c. $2x = -6$	c. _____
d. $x = -3$	d. _____

**Practice****Preparing for Two-Column Proofs****Name the property or equality that justifies each statement.**

1. If  $m\angle A = m\angle B$ , then  $m\angle B = m\angle A$ .
2. If  $x + 3 = 17$ , then  $x = 14$ .
3.  $xy = xy$
4. If  $7x = 42$ , then  $x = 6$ .
5. If  $XY - YZ = XM$ , then  $XM + YZ = XY$ .
6.  $2(x + 4) = 2x + 8$
7. If  $m\angle A + m\angle B = 90$ , and  $m\angle A = 30$ , then  $30 + m\angle B = 90$ .
8. If  $x = y + 3$  and  $y + 3 = 10$ , then  $x = 10$ .

**Complete each proof by naming the property that justifies each statement.**

9. Prove that if
- $2(x - 3) = 8$
- , then
- $x = 7$
- .

**Given:**  $2(x - 3) = 8$ **Prove:**  $x = 7$ **Proof:**

Statements	Reasons
a. $2(x - 3) = 8$	a. _____
b. $2x - 6 = 8$	b. _____
c. $2x = 14$	c. _____
d. $x = 7$	d. _____

10. Prove that if
- $3x - 4 = \frac{1}{2}x + 6$
- , then
- $x = 4$
- .

**Given:**  $3x - 4 = \frac{1}{2}x + 6$ **Prove:**  $x = 4$ **Proof:**

Statements	Reasons
a. $3x - 4 = \frac{1}{2}x + 6$	a. _____
b. $\frac{5}{2}x - 4 = 6$	b. _____
c. $\frac{5}{2}x = 10$	c. _____
d. $x = 4$	d. _____

## Reading to Learn Mathematics

## Preparing for Two-Column Proofs

## Key Terms

**two-column proof** a deductive argument that contains statements and reasons organized in two columns

## Reading the Lesson

- State whether each statement is *true* or *false*. If the statement is false, explain why.
  - Algebraic properties can be used as reasons in proofs.
  - When you solve an equation, you are using inductive reasoning.
  - In a two-column proof, you must give a reason for each statement.
  - The last statement in a two-column proof is the given information.

- Fill in the missing statements and reasons in the two-column proof.

**Given:**  $a \parallel b$ ,  $c \parallel d$

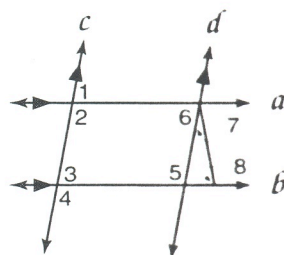
**Prove:**  $m\angle 2 = m\angle 7 + m\angle 8$

**Proof:**

**Statements**

**Reasons**

a. _____	a. Given
b. $\angle 2 \cong \angle 4$	b. _____
c. $\angle 4 \cong \angle 5$	c. _____
d. _____	d. Transitive Property of Congruence
e. $m\angle 2 = m\angle 5$	e. _____
f. $m\angle 5 = m\angle 7 + m\angle 8$	f. _____
g. _____	g. Substitution Property of Equality



## Helping You Remember

- A good way to remember some terms is to compare them. Write several sentences comparing the similarities and differences between paragraph proofs and two-column proofs.